A new method for synthesizing multiple-period adaptive–repetitive controllers and its application to the control of hard disk drives[☆]

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A B S T R A C T

This paper introduces a new method for synthesizing multiple-period repetitive controllers. The main innovations in the synthesis procedure presented in this article are two. The first one is that this technique yields a solution compatible with the integration of the computed multiple-period repetitive controller into a minimum-variance adaptive control scheme. The second innovation is that the solution is period-recursive, reducing the complexity of controller synthesis considerably when compared with other methods available in the literature. To exemplify the synthesis procedure, a multiple-period repetitive controller is designed and integrated into an adaptive–repetitive control scheme used in the track-following control of a commercial hard disk drive. Experimental results show the effectiveness of the presented approach.

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1. Introduction

Repetitive control (Hara, Yamamoto, Omata, & Nakano, 1998; Inoue, Nakano, Kubo, Matsumoto, & Baba, 1981; Tomizuka, Tsao, & Chew, 1989) has been demonstrated to be very effective in rejecting disturbances when implemented on systems affected by periodic disturbances, such as, *hard disk drives* (HDD), electric motors and generators, other rotating machines, and satellites (Broberg & Molyet, 1992a,b; Liang, Green, Weiss, & Zhong, 2002; Longman, Yeol, & Ryu, 2006; Pérez Arancibia, Lin, Tsao, & Gibson, 2007a; Senjyu, Miyazato, & Uezato, 1995; Yamada, Riadh, & Funahashi, 1999). Also, repetitive control has been shown to be an appropriate tool when applied to periodic tracking problems in power electronics, manufacturing and robotics (Cosner, Anwar, & Tomizuka, 1990; Costa-Castelló, Griñó, & Fossas, 2004; Ratcliffe,

Hätönen, Lewin, Rogers, & Owens, 2006; Tsai, Anwar, & Tomizuka, 1988; Zhou & Wang, 2003; Zhou et al., 2007). In both kinds of problems, disturbance rejection and tracking, it is not rare to encounter applications where controllers capable of dealing with signals composed of multiple periods are required. Common examples are electromechanical systems containing multiple gears. This paper is devoted to the development of a new method for synthesizing repetitive controllers capable of rejecting multi-periodic output disturbances affecting the plant to be controlled.

The main feature of the method introduced here is that it produces multiple-period controllers suitable for integration into the combined adaptive–repetitive control scheme presented in Pérez Arancibia et al. (2007a), which is based on the notions of *internal model* (Francis & Wonham, 1976) and *adaptive minimum-variance regulation* (Horowitz, Li, & McCormick, 1998). The first part of this paper deals with the reformulation of the original disturbance rejection control problem as a polynomial algebraic one, and also, with finding an explicit analytical solution for it. In general, the existence of a solution with an explicit analytical expression does not guarantee simple computability. For this reason, the second part of this paper presents the development of a recursive algorithm that reduces significantly the complexity of control synthesis.

Previous works have addressed the problem of multiple-period repetitive control, from both theoretical and practical perspectives, (e.g., Garimella & Srinivasan, 1996; Krishnamoorthy & Tsao, 2005; Owens, Li, & Banks, 2004; Owens, Tomas-Rodriguez, Hatönen, & Li, 2006; Yamada et al., 1999; Yamada, Riadh, & Funahashi,
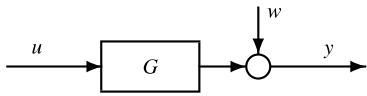
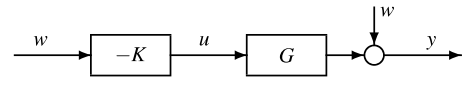**Fig. 1.** LTI plant G and output disturbance w.

Fig. 2. Feedforward output disturbance rejection scheme.

2000). However, those solutions are not easily integrable into the scheme presented in Pérez Arancibia et al. (2007a), considered here. For that reason, in this work we introduce an alternative approach, which extends the methods for designing one-period adaptive–repetitive controllers in Pérez Arancibia et al. (2007a) to the multi-periodic case, following the ideas and guidelines in Åström and Wittenmark (1984), Tomizuka (1987), Tomizuka et al. (1989) and Tsao and Tomizuka (1994). Experimental results obtained using a commercial HDD demonstrate the effectiveness of the resulting control synthesis method.

The rest of the paper is organized as follows. Section 2 reviews some fundamentals concepts of repetitive control. Section 3 presents the main contribution of this paper, which is a new method for synthesizing multiple-period repetitive controllers. Section 4 describes the multiple-period adaptive–repetitive control scheme to which the controller solution in Section 3 can be integrated to. Section 5 presents experimental results. Finally, some conclusions are given in Section 6.

Notation.

- z^{-1} denotes the delay operator, i.e., for a signal x, $z^{-1}x(t) = x(t-1)$ and conversely $zx(t) = x(t+1)$. Notice that since some of the systems involved in this paper are time-varying, here, z is not necessarily the complex variable associated to the z-transform.
- RH_∞ denotes the set containing all the LTI systems that are rational and stable as defined in Zhou, Doyle, and Glover (1996).
- $\| \cdot \|_2$ denotes the standard H_2 norm of a LTI system.
- $\| \cdot \|_\infty$ denotes the standard H_∞ norm of a LTI system.
- $| \cdot |$ denotes the standard module of a complex number.
- The upper index $^{(n)}$ is used to denote the recursion number n in a recursive algorithm. This does not denote an exponent.
- For a generic discrete random process $\mathbf{y}$, a realization of $\mathbf{y}$ is denoted by y.
- $\mathbb{N}$ denotes the set of positive integer numbers. $\mathbb{R}$ denotes the set of real numbers.

2. Preliminaries on repetitive control

2.1. Repetitive control for disturbance rejection

In this section, we review some fundamental ideas on one-period repetitive control that will be used later in this paper. First, consider the block diagram in Fig. 1. There, G is a stable LTI system and w is a disturbance considered to be mostly formed by a combination of sinusoidal sequences with frequencies multiple of a fundamental one. If the original plant system is unstable, it is assumed that it can be stabilized by LTI feedback control.

To begin with, we describe a repetitive control method for feedforward disturbance rejection in which the signal w is assumed to be available for measurement. This is a design assumption, since in practice w can be estimated but not directly measured. Also, it is assumed that the fundamental frequencies of the periodic signals forming part of w are a priori known. Thus, the natural control goal is the synthesis of a stable feedforward filter K, such that, the frequency response of the LTI system $1-KG$ is zero at the same periodic frequencies of the sinusoidal signals composing w. This approach results in the block diagram in Fig. 2, where

$$y = w - GKw = (1 - GK)w. \tag{1}$$

Notice that the problem posed as in Fig. 2 becomes a feedforward tracking control problem. It is immediate that for the ideal case where G is minimum phase with relative degree 0, the best choice is to pick $K = G^{-1}$. However, it is not unusual to encounter discrete-time systems, obtained from sample-and-hold equivalence of continuous-time systems, that have unstable zeros. Thus, as in Tsao (1994), a possible design choice is to select a desired model M, and then find a minimizing K of some system norm of $M - GK$, for example, the H_∞ norm or the H_2 norm. Another option, the one chosen here as in Pérez Arancibia et al. (2007a), is to define an error transfer function $E = 1 - GK$ and then force the frequency response of E to be zero periodically at certain desired frequencies. This objective is achievable by using the polynomial design methods in Åström and Wittenmark (1984), following the general guidelines presented in Tomizuka et al. (1989) and Tomizuka (1987). The main idea is to enforce an error transfer function with the form $E = RD$, where D can be thought of as an internal model for the disturbance w, and R is an a priori unknown stable transfer function. For the one-periodic class of signals considered in this section, the internal model is chosen to be

$$D = 1 - qz^{-N}, \tag{2}$$

where q is a zero-phase low-pass filter and N is the period of the periodic disturbance to be attenuated.

The filter q will allow us some flexibility over the frequency range of disturbances to be canceled while maintaining stability. The filter D has a combed shape with notches matching the frequencies of the periodic signals forming part of w. Thus, a filter K that makes the frequency response of E zero at desired periodic frequencies can be computed by solving the Diophantine equation

$$RD + KG = 1, \tag{3}$$

where R and K are the unknowns.

Now, we briefly discuss the existence of solutions for (3). First, notice that (3) can be rearranged as

$$b_R (a_K a_G b_D) + b_K (a_R a_D b_G) = a_K a_G a_R a_D, \tag{4}$$

where the polynomial numerators are denoted by the symbol b, the polynomial denominators by the symbol a and the sub-indices indicate the corresponding transfer function in (3). It is immediate from Åström and Wittenmark (1984) and references therein, e.g., Kučera (1979), that for given polynomials a_G, b_D, a_D and b_G and chosen polynomials a_K and a_R, (4) has a solution if and only if the greatest common factor of $a_K a_G b_D$ and $a_R a_D b_G$ divides $a_K a_G a_R a_D$. In general if this condition is satisfied, we say that G and D are coprime.

As shown in Tomizuka et al. (1989) if a solution pair $\{R_o, K_o\}$ is found, then (3) characterizes a whole family of stabilizing internal model based repetitive controllers. As in Pérez Arancibia et al. (2007a), following the guidelines in Tomizuka (1987) and Tomizuka et al. (1989) a method for finding a particular solution pair $\{R_o, K_o\}$ is presented here. The general methodology of Tomizuka (1987) and Tomizuka et al. (1989) is also employed in Yamada et al. (2000) in the context of multiple-period repetitive control. The method starts by separating G into its minimum and non-minimum phase parts, denoted by G_+ and G_- respectively. Thus,

$$G = \frac{B}{A} = \frac{B_+ B_-}{A} = G_+ G_-,$$

$$G_+ = \frac{B_+}{A}, \qquad G_- = B_-. \tag{5}$$
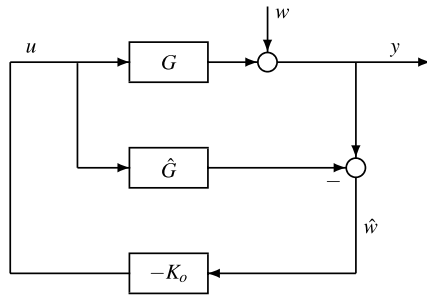
Fig. 3. Estimation of w and repetitive control scheme. G: true plant; $\hat{G}$: identified model of G; K_o: repetitive controller; w: output disturbance; $\hat{w}$: online estimate of w.

Where all the zeros of B_+ are stable, and all the zeros of B_- are unstable. Often, B_+ and B_- are referred as the cancelable and uncancelable parts of the numerator B of G, respectively. Now, substituting (5) into (3) we can write

$$RD + \kappa G_- = 1, \quad \kappa = KG_+. \tag{6}$$

Among the infinity many solutions to (6) it is verifiable by simple algebraic manipulations that one of the solutions is given by

$$R_o = \frac{1}{1 - (1 - \gamma G_-^* G_-) q z^{-N}}, \tag{7}$$

$$\kappa_o = q \gamma G_-^* z^{-N} R_o, \qquad K_o = \kappa_o G_+^{-1}.$$

Here, G_-^* is defined as $G_-^*(z^{-1}) = G_-(z)$, and $0 < \gamma \in \mathbb{R}$. From this point onwards, in block diagrams and equations we employ the symbols K_o and R_o, under the understanding that those correspond to the specific solution in (7).

The block diagram in Fig. 2 assumes that the signal disturbance w is available for measurement. However, in practice w has to be estimated online according the diagram in Fig. 3, where $\hat{w}$ is an estimate of w and $\hat{G}$ is an identified model for the plant G. Clearly, when the estimation scheme shown in Fig. 3 is employed, the feedforward disturbance cancelation filter K_o becomes part of a feedback controller υ_o computable as

$$\upsilon_o = \frac{-K_o}{1 - K_o \hat{G}}. \tag{8}$$

It is immediately clear that being υ_o a SISO LTI controller, its stability and performance can be analyzed using all the tools of classical control, such as, gain and phase margins, along with the use of sensitivity functions. In this case, we are interested on the output disturbance sensitivity function

$$\zeta_o = \frac{1}{1 - G\upsilon_o}. \tag{9}$$

Notice that under the assumption $\hat{G} = G$

$$\zeta_o = 1 - GK_o, \tag{10}$$

which is the feedforward mapping from w to y shown in Fig. 2. This implies that the closed-loop performance is not altered by the estimation process, as long as, the model $\hat{G}$ is an exact representation of the true plant G.

The previous development establishes that the closed-loop system will be stable for stable plants G and K_o under the assumption that $\hat{G} = G$. Thus, we need a method to ensure that the design algorithm produces a stable K_o. This is discussed in the next subsection.
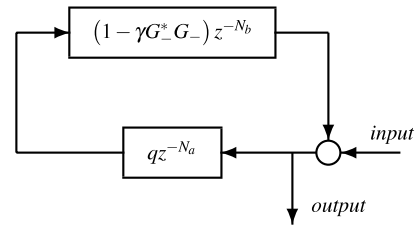
Fig. 4. Representation of R_o as a typical feedback configuration.

2.2. Nominal stability analysis

In order to have a nominally stable closed-loop system, all what needs to be done is to ensure that the design algorithm yields a stable controller K_o. Recalling the definition of K_o in (7), notice that K_o is formed by the multiplication of the systems $q\gamma G_-^* z^{-N}$, R_o, and G_+^{-1}. By definition G_+^{-1} is stable. The system $q\gamma G_-^* z^{-N}$ is stable and causal provided that q is stable and that N is large enough. Thus, K_o will be stable as long as R_o is stable. Here, we state a sufficient condition for the stability of R_o, which is based on the *small gain theorem* (Dahle & Diaz-Bobillo, 1995). First, notice that R_o can be represented by the block diagram in Fig. 4, where $N = N_a + N_b$, such that, z^{-N_a} and z^{-N_b} make the systems qz^{-N_a} and $\left(1 - \gamma G_-^* G_-\right) z^{-N_b}$ causal respectively.

The small gain theorem implies that a sufficient condition for asymptotic stability is

$$\left\| \left(1 - \gamma G_-^* G_-\right) z^{-N_b} \right\|_\infty \left\| q z^{-N_a} \right\|_\infty < 1, \tag{11}$$

which can be translated into

$$|1 - \gamma G_-^*(e^{j\theta}) G_-(e^{j\theta})| < \frac{1}{|q(e^{j\theta})|}, \quad \forall \theta \in [0, \pi]. \tag{12}$$

In (12) the real number γ can be thought of as a stability and performance tuning parameter. It is important to emphasize that when (12) is satisfied, the resulting controller K_o is stable and consequently the closed-loop system is stable as well. The systems involved are LTI, therefore, an appropriate way to analyze stability robustness is the use of the notions of gain and phase margins, from classical control theory. This approach is employed in the design examples to be presented later in this paper.

3. Multiple-period repetitive control

3.1. Proposed controller design method

In this subsection we introduce the main contribution of this paper, which is a new method for synthesizing multiple-period repetitive controllers. The design method for synthesizing one-period repetitive controllers presented in the previous section can be integrated into a combined adaptive–repetitive controller scheme to be presented in Section 4. In order to design a multiple-period adaptive–repetitive controller with the same capability, we need to solve (6) with a corresponding multiple-period internal model D. From the internal model principle it follows that for targeting L different periods, a suitable L-period D is given by

$$D = \prod_{k=1}^{L} 1 - q_k z^{-N_k}. \tag{13}$$

Thus, the next step is to solve (6) with the internal model in (13). In order to accomplish that, first we need to define some concepts to be used later.

Definition 1. An *r-combination* of a set is a subset of size r. For example, for the set $\{a, b, c\}$, we have the following three 2-combinations: $\{a, b\}$, $\{a, c\}$ and $\{b, c\}$.

Definition 2. For a LTI system G_-, exponentials are defined as follows: $G_-^0 = 1$ and $G_-^k = G_- G_- \cdots G_-$, k times.

Definition 3. For a number $k \in \mathbb{N}$, the system f_k is defined as

$$f_k = \frac{G_-^* q_k \gamma_k z^{-N_k}}{1 - (1 - \gamma_k G_-^* G_-) q_k z^{-N_k}}, \tag{14}$$

where q_k is a zero-phase low-pass filter, $0 < \gamma_k \in \mathbb{R}$, and $N_k \in \mathbb{N}$.

With the use of the previous definitions we state a theorem that will serve as a guideline for synthesizing multiple-period repetitive controllers.

Theorem 1. *A particular solution to the L-period problem is given by*

$$R_o = \prod_{k=1}^{L} \frac{1}{1 - (1 - \gamma_k G_-^* G_-) q_k z^{-N_k}} \tag{15}$$

and by

$$\kappa_o = \sum_{k=1}^{L} \sum_{s \in S_k} (-1)^{k+1} G_-^{k-1} F_{ks}, \qquad K_o = \kappa_o G_+^{-1}, \tag{16}$$

with

$$F_{ks} = \prod_{j=1}^{k} f_{s_j}, \tag{17}$$

where s is a vectorial index $s = \{s_1, \ldots, s_k\} \in \mathbb{N}^k$ and S_k is the set that contains all the k-combinations of the set $\{1, 2, \ldots, k, \ldots, L\}$, and the functions f_{s_j} are computed according to (14).

Proof. Notice that relation (16) holds if and only if the system $\kappa_o^{(n+1)}$ for the $(n+1)$-period case can be computed recursively from the system $\kappa_o^{(n)}$ for the n-period case as

$$\kappa_o^{(n+1)} = \kappa_o^{(n)} - \kappa_o^{(n)} G_- f_{n+1} + f_{n+1}, \tag{18}$$

with $\kappa_o^{(1)}$ given by (7). Notice that the superscripts in parentheses, $(n + 1)$, (n), etc., refer to the recursion number and they do not denote exponentials as in Definition 2.

Also, it is immediate that

$$R_o^{(n+1)} = \frac{R_o^{(n)}}{1 - (1 - \gamma_{n+1} G_-^* G_-) q_{n+1} z^{-N_{n+1}}}, \tag{19}$$

with $R_o^{(1)}$ given by (7). Thus, having the solution given by (18) and (19) a recursive form, this is proven using mathematical induction.

- For $n = 1$: It follows immediately from (7).
- For $n + 1$ assuming the solution for n: What needs to be shown is that relations (18) and (19) satisfy

$$R_o^{(n+1)} D^{(n+1)} + \kappa_o^{(n+1)} G_- = 1, \tag{20}$$

provided that

$$R_o^{(n)} D^{(n)} + \kappa_o^{(n)} G_- = 1. \tag{21}$$

To show that (18), (19) and (21) imply (20), the right side of (18) and the right side of (19) are replaced into the left side of (20). Thus, we obtain

$$\frac{R_o^{(n)} D^{(n)} + \kappa_o^{(n)} G_-}{1 - (1 - \gamma_{n+1} G_-^* G_-) q_{n+1} z^{-N_{n+1}}} - \frac{(R_o^{(n)} D^{(n)} + \kappa_o^{(n)} G_-) q_{n+1} z^{-N_{n+1}} - \gamma_{n+1} G_-^* G_- q_{n+1} z^{-N_{n+1}}}{1 - (1 - \gamma_{n+1} G_-^* G_-) q_{n+1} z^{-N_{n+1}}}. \tag{22}$$

Now, noticing (21), it follows that the numerator and denominator in (22) are identical, then (20) follows, and therefore, (15) and (16) follow as well, which completes the proof of Theorem 1. $\square$

Remark 1. It is important to remark that this proof comprises the two main contributions of the paper. The first one is that this mathematical proof shows that the proposed pair $\{R_o, K_o\}$ in (15), (16) and (17) is in fact a particular solution to the Diophantine equation in (3) for the internal model D in (13). The second one is that being the proof constructive, relation (18) gives us a recursive method for synthesizing controllers for an arbitrary number of periods. From a practical viewpoint this is very important, since otherwise, the synthesis and experimental tuning of multiple-period repetitive controllers would be extremely difficult due to the great number of parameters that would be necessary to choose simultaneously.

In the following paragraphs we show examples aimed to explain the controller synthesizing process.

Example 1. Consider a generic case with $L = 3$. The computation of $R_o^{(3)}$ is immediate from (15). The computation of $\kappa_o^{(3)}$ is done as follows.

To begin with, we compute the systems defined by (17). For $k = 3$, the only 3-*combination* of the set $\{1, 2, 3\}$ is $\{1, 2, 3\}$, then $S_3 = \{\{1, 2, 3\}\}$, and therefore, we have

$$F_{3\{1,2,3\}} = f_1 f_2 f_3.$$

For $k = 2$, the 2-*combinations* of the set $\{1, 2, 3\}$ are $\{1, 2\}$, $\{1, 3\}$ and $\{2, 3\}$, then $S_2 = \{\{1, 2\}; \{1, 3\}; \{2, 3\}\}$, and therefore, we have

$$F_{2\{1,2\}} = f_1 f_2; \qquad F_{2\{1,3\}} = f_1 f_3; \qquad F_{2\{2,3\}} = f_2 f_3.$$

For $k = 1$, the 1-*combinations* of the set $\{1, 2, 3\}$ are $\{1\}$, $\{2\}$ and $\{3\}$, then $S_1 = \{\{1\}; \{2\}; \{3\}\}$, and therefore, we have

$$F_{1\{1\}} = f_1; \qquad F_{1\{2\}} = f_2; \qquad F_{1\{3\}} = f_3.$$

Now, we compute the interior sums in (16). For $k = 3$, the interior sum is

$$\sum_{s \in S_3} F_{3s} = f_1 f_2 f_3.$$

For $k = 2$, the interior sum is

$$\sum_{s \in S_2} F_{2s} = f_1 f_2 + f_1 f_3 + f_2 f_3.$$

For $k = 1$, the interior sum is

$$\sum_{s \in S_1} F_{1s} = f_1 + f_2 + f_3.$$

Thus, $\kappa_o^{(3)}$ is given by

$$\kappa_o^{(3)} = f_1 + f_2 + f_3 - G_-(f_1 f_2 + f_1 f_3 + f_2 f_3) + G_-^2 f_1 f_2 f_3.$$

Remark 2. Notice that given that the vectorial indices s are formed using the concept of r-combination, the order of the elements in s is irrelevant, and therefore, permutations of the elements in s do not change the index. For example, consider S_2 in Example 1. There, if the second index in S_2, $\{1, 3\}$, is replaced by $\{3, 1\}$, the result remains invariant since $F_{2\{3,1\}} = f_3 f_1 = f_1 f_3 = F_{2\{1,3\}}$.

As stated in Remark 1, relation (18) gives us a method for synthesizing repetitive controllers with any number of periods

recursively. This is illustrated with the following example.

Example 2. We consider the same problem that in Example 1, but this time, we solve it recursively. To begin with, notice that

$$\kappa_o^{(1)} = f_1.$$

Then, the second recursion is given by

$$\kappa_o^{(2)} = f_1 - f_1 G_- f_2 + f_2.$$

And finally, the third recursion solves the problem as

$$\kappa_o^{(3)} = f_1 - f_1 G_- f_2 + f_2 - (f_1 - f_1 G_- f_2 + f_2) G_- f_3 + f_3$$
$$= f_1 + f_2 + f_3 - G_- (f_1 f_2 + f_1 f_3 + f_2 f_3) + G_-^2 f_1 f_2 f_3.$$

3.2. Nominal stability analysis

The first thing to notice is that the controller K_o is formed by summations and products of transfer functions f_k, $k = 1, \ldots, L$ with themselves and with the plant G_-. Therefore, if each transfer function in the set $\{f_1, f_2, \ldots, f_L\}$ is stable, then the resulting controller K_o will be stable as well. Thus, following the development in the previous section, it is immediate that a sufficient condition for stability is

$$\left| 1 - \gamma_k G_-^* (e^{j\theta}) G_- (e^{j\theta}) \right| < \frac{1}{\left| q_k (e^{j\theta}) \right|}, \quad \forall \theta \in [0, \pi]$$
$$\text{for } k = 1, \ldots, L. \tag{23}$$

This condition might look conservative. However, in the experimental section, we show that this is an appropriate guideline for design.

4. An adaptive–repetitive control scheme

4.1. Repetitive and minimum-variance control

The principal reason for solving the repetitive control problem as in the previous sections is that this method allows us to formulate a H_2 control problem, which can be approximated by the adaptive scheme introduced in Pérez Arancibia et al. (2007a), described here. To begin with, let us consider an arbitrary rational LTI asymptotically stable filter Q, i.e., $Q \in RH_\infty$. Also, let

$$R(Q) = R_o - QG, \tag{24}$$
$$K(Q) = K_o + QD. \tag{25}$$

It is clear that systems $R(Q)$ and $K(Q)$ in (24) and (25) define an entire family of solutions to the Diophantine equation in (3). Notice that $R(Q)$ and $K(Q)$ belong to RH_∞ for all $Q \in RH_\infty$, provided that D and G are stable.

The previous parametrization allows us to formulate a new control problem as an optimization one. Specifically, we would like to minimize the variance of the system output random variable $\mathbf{y}(k) \forall k$. Notice that from this perspective, the sequence y, in Fig. 2 and other figures, is a realization of the random process $\mathbf{y}$. Now, let $\mathbf{y}$ be a stationary mean-ergodic and covariance-ergodic random process for any given stable LTI filter Q. Then, the problem becomes

$$\min_{Q \in RH_\infty} E\{\mathbf{y}^2(k)\}. \tag{26}$$

Notice, that if $E\{\mathbf{y}^2(k)\} = \sigma^2$, the ergodicity assumption implies that $\lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} y^2(k) = \sigma^2$, with probability 1. Also, it is verifiable that (26) is equivalent to the H_2 problem
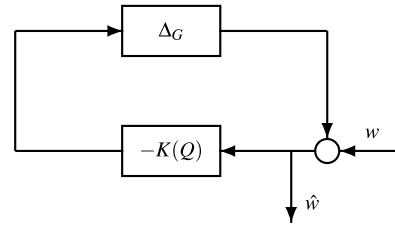
$$\min_{Q \in RH_\infty} \|W - GK(Q)W\|_2, \tag{27}$$

Fig. 5. Systems Δ_G and $K(Q)$ in typical feedback configuration.

where W is a stable filter that maps a stationary, white, zero-mean, unit-variance random sequence to the disturbance w. Filters like W are usually called disturbance models of w. Considering (3) and the parameterized systems $K(Q)$ and $R(Q)$, (27) is equivalent to

$$\min_{Q \in RH_\infty} \|R_o DW - QGDW\|_2. \tag{28}$$

It is important to remark that the solution to (27) requires a disturbance model W. In practice the identification of an accurate model W is extremely difficult and often impossible, and also, it is not clear how a solution to (27) can be adaptively approximated. Fortunately, (28) can be approximated with the use of adaptive filters. This is key in the development of the adaptive–repetitive control scheme to be introduced later in this section.

4.2. Stability of the closed-loop system with $K(Q)$

Similarly to the case studied in Section 2, if the controller K_o in Fig. 3 is replaced by $K(Q)$, then under the assumption that $\hat{G} = G$, the sensitivity function from the disturbance w to the output y is given by

$$\zeta_{K(Q)} = 1 - GK(Q). \tag{29}$$

This implies that for stable systems K_o, D and Q, the closed-loop system is nominally stable. Clearly, for any LTI Q, the stability robustness of the closed-loop system can be analyzed using classical indices, such as, gain and phase margins.

For reasons that will become clear in the next subsection, many times it results useful to look at the stability problem from an alternative perspective. For that purpose, let us consider plant additive uncertainty, i.e.,

$$G = \hat{G} + \Delta_G, \tag{30}$$

and then, replace K_o by the new controller $K(Q)$. Thus, the block diagram in Fig. 3 is equivalent to the block diagram in Fig. 5, which implies that, invoking the small gain theorem, a sufficient stability condition is given by

$$\|\Delta_G\|_\infty \|K(Q)\|_\infty < 1. \tag{31}$$

It is worth mentioning that this condition is consistent with the feedforward stability condition based on (29), since for the case $\hat{G} = G$, the system is always stable.

Thus far, we have assumed that Q is a stable LTI system, however, it is important to note that for the case in which Q is time-varying, the condition remains essentially the same, except for the replacement of the H_∞ norm ($\| \cdot \|_\infty$) by a chosen induced norm. The ℓ_2-induced norm ($\| \cdot \|_{\ell_2 \to \ell_2}$) or the ℓ_∞-induced norm ($\| \cdot \|_{\ell_\infty \to \ell_\infty}$), for example.

4.3. Proposed adaptive scheme

The solution to (28) can be found employing well-known H_2 control methods (Zhou et al., 1996). However, in order to apply those methods we would need a reliable model W capable of capturing all the relevant statistical information contained in the disturbance signal w. A system W can be identified using some identification method. For example, in Pérez-Arancibia, Tsao, and Gibson (2010) disturbance models are identified using the *n4sid* subspace method in Van Overschee and De Moor (1996).

In most applications the identification of disturbance models is challenging and often times impossible. For this reason, it is convenient to translate the problem in (28) into an adaptive filtering problem, solvable online by the use of algorithms such as RLS (*recursive least-squares*) or LMS (*least-mean-squares*). In this case, the standard LMS algorithm and the inverse QR-RLS algorithm in Sayed (2003) are employed to demonstrate the proposed method in experiments. The proposed adaptive scheme is shown in Fig. 6, where the controller $K(Q) = K_o + QD$ can be broken into a repetitive part, K_o, and an adaptive part, QD.

The fundamental idea behind the scheme in Fig. 6 is that the adaptive algorithm is run using a regressor formed by values from the signal Dw, and not w, as in the typical minimum-variance adaptive configurations. Thus, the periodic content to be canceled in w is attenuated by K_o, and what is left, Dw, is attenuated adaptively. In the experiments presented in this paper, we introduce the constraint $Q(z) = \sum_{i=0}^{N_Q} \theta_i z^{-i}$, where N_Q is the order of the filter Q and $\theta_i \in \mathbb{R}$. This allows us to enforce the stability of Q, since *finite impulse response* (FIR) filters are always stable provided that the coefficients remain bounded.

The stability arguments given in the previous section, for the case when Q is LTI, can be extended to the case when Q is time-
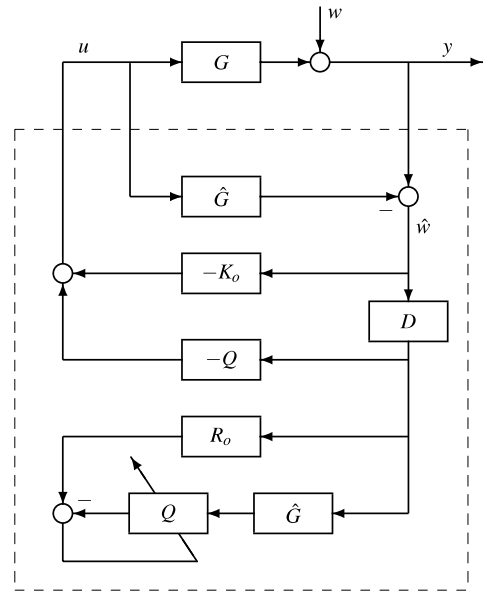

Fig. 6. Adaptive–repetitive control scheme.

varying. Notice that under the assumption that $\hat{G} = G$, the system in Fig. 6 will be ℓ_2-stable for any ℓ_2-stable Q and ℓ_∞-stable for any ℓ_∞-stable Q. Similarly to the LTI case, if additive uncertainty is assumed as in (30), the system in Fig. 6 will remain ℓ_2-stable (or ℓ_∞-stable) as long as the ℓ_2-induced (or correspondingly the ℓ_∞-induced) norm of $K(Q)$ remains small enough. Noticing that the adaptive system in Fig. 6 falls in the category of controllers known as adaptive–inverse schemes in Widrow and Walach (1996)
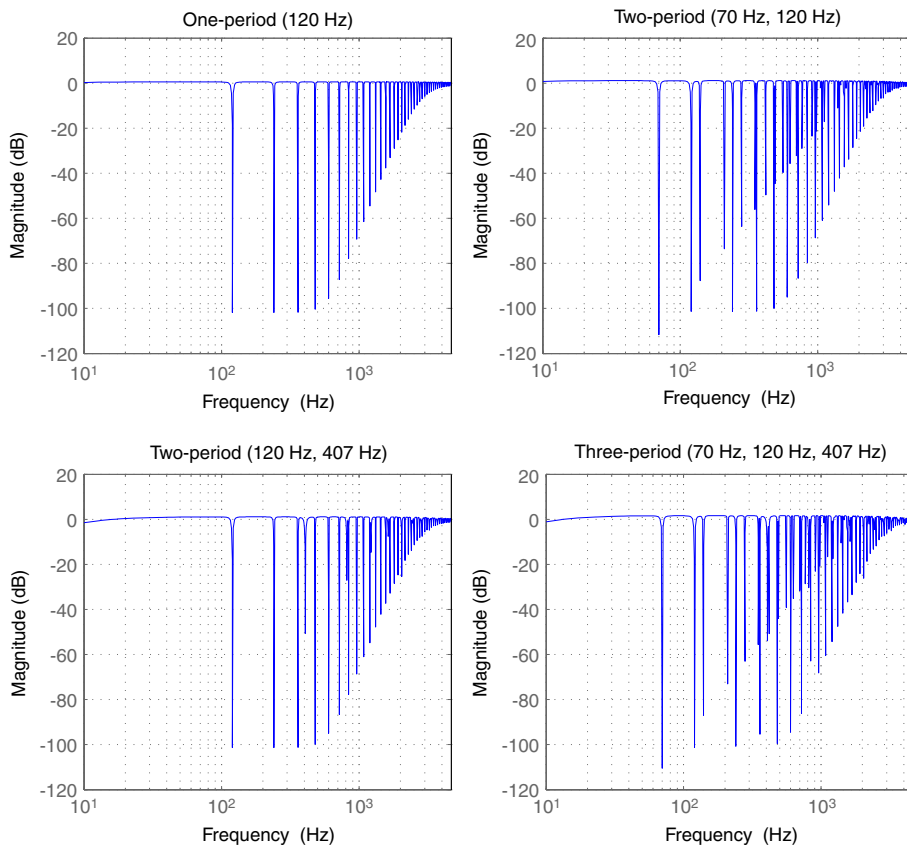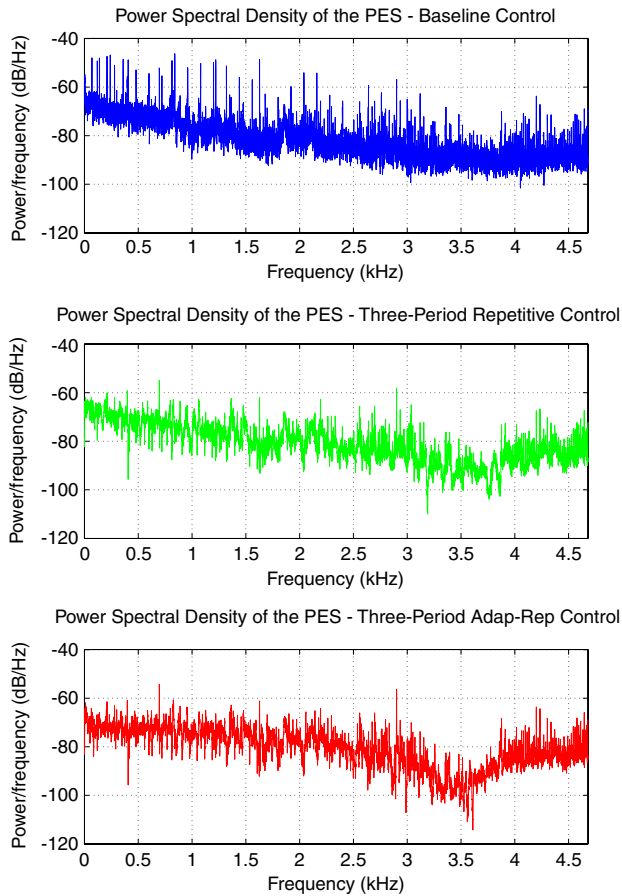

Fig. 7. Sensitivity function ζ_o for the cases: one-period 120 Hz; two-period 70, 120 Hz; two-period 120, 407 Hz; and three-period 70, 120, 407 Hz.

Fig. 8. Power spectral density of the PES *y* for three different experiments. *Upper Plot*: baseline control. *Middle Plot*: three-period repetitive control (70, 120, 407 Hz). *Bottom Plot*: three-period adaptive–repetitive control (70, 120, 407 Hz). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and Widrow and Stearns (1985), and as self-tuning regulators in Horowitz et al. (1998) and Horowitz and Li (1996), it is possible to extend the stability and convergence analyses presented in those works to the case in Fig. 6, using the available adaptive control and filtering theory in textbooks (Goodwin & Sin, 1984; Ioannou & Fidan, 2006; Ljung & Söderström, 1983; Sayed, 2003; Widrow & Walach, 1996).
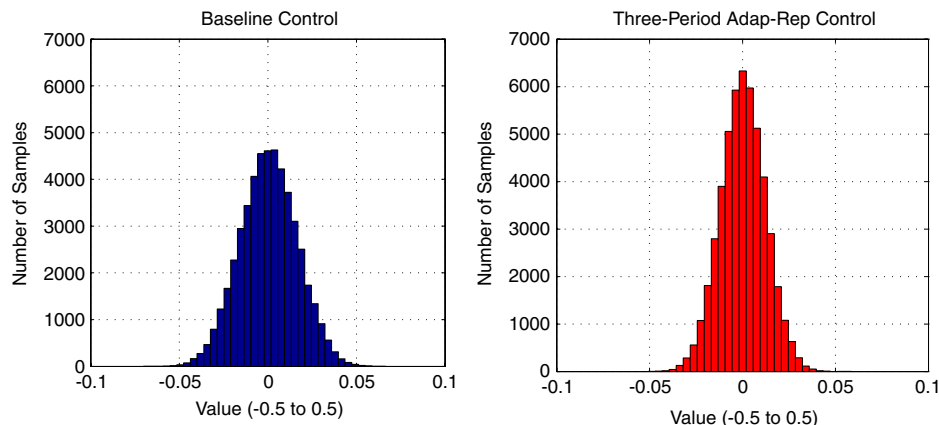
Table 1
Experimental Results: 3σ index obtained with the adaptive–repetitive scheme at Head 1, Track 20,000 (baseline $3\sigma = 4.9896$).

Algorithm/order of Q	1-period	2-period	3-period
No filter Q	4.4176	4.2805	3.9557
Inverse QR-RLS/16	3.9218	4.0497	3.8292
Inverse QR-RLS/64	3.7396	3.9122	3.6979
LMS/16	4.1380	4.1275	3.9009
LMS/64	4.0797	4.0791	3.8972
LMS/128	3.9617	4.0454	3.8903
LMS/256	3.9362	4.0157	3.8871

5. Experiments

5.1. Description of the experiment

The experimental effectiveness of the proposed control scheme is demonstrated using a commercial HDD system. The description of the HDD system and the details of the experimental implementation are discussed in Pérez-Arancibia et al. (2010). Generally speaking, a HDD is a mechatronic device that uses rotating platters to store data. Information is recorded on and read from concentric cylinders or tracks by read–write magnetic transducers called heads that fly over the magnetic surfaces of the HDD platters. The position of the heads over the platters is changed by an actuator that consists of a coil attached to a link, which pivots about a ball bearing. This actuator connects to the head by a steel leaf called suspension (Abramovitch & Franklin, 2002; Messner & Ehrlich, 2001).

The control objective is to position the center of the head over the center of a data track. Thus, the typical measure of HDD tracking performance is the deviation of the center of the head from the center of a given track, which is often called *track misregistration* (TMR) (Messner & Ehrlich, 2001). A common index to quantify TMR is 3σ, where σ is the empirical standard deviation of the *position error signal* (PES). It is common to express 3σ as a percentage of the track pitch (Messner & Ehrlich, 2001), which must be less than 10% in order to be considered acceptable. TMR values larger than this figure will produce excessive errors during the reading and recording processes. In the experiments presented here we used a 2-platter (10 GB/platter) 4-head 7200 rpm commercial HDD and a *Mathworks* ® xPC Target system for control with a sample-and-hold rate of approximately 9.36 kHz.

In the experiments to be described in the last subsection of the paper, we use a combination of two LTI controllers, developed and implemented in Pérez-Arancibia et al. (2010) and Pérez-Arancibia, Tsao, and Gibson (2007b), as a baseline for adding the

Fig. 9. Histogram comparing the experimental PES *y*, obtained with the use of the baseline controller and with the use of the multiple-period adaptive–repetitive control scheme (70, 120, 407 Hz).

Table 2
Performance index 3σ of the *Position Error Signal* (PES) as percentage of the track width.

	Head 0			Head 1		
	Track 10,000	Track 15,000	Track 20,000	Track 10,000	Track 15,000	Track 20,000
Baseline controller	5.1126	5.0913	4.9632	5.1097	5.2278	4.9896
1-period repetitive control	4.3546	4.2957	4.1964	4.3157	4.2970	4.4176
2-period repetitive control	4.4031	4.1823	4.2438	4.2381	4.1765	4.2805
3-period repetitive control	4.1795	3.9186	4.0489	4.0646	3.9709	3.9557
Adapt.-rep.—1-period (inv. QR-RLS/64)	3.9585	3.8223	3.8255	3.7699	3.9311	3.7396
Adapt.-rep.—2-period (inv. QR-RLS/64)	4.1138	3.9149	4.0257	3.9616	3.9473	3.9122
Adapt.-rep.—3-period (inv. QR-RLS/64)	3.9814	3.7126	3.8097	3.7278	3.7546	3.6979
Adapt.-rep.—1-period (LMS/256)	4.1694	4.0317	4.1034	4.0438	3.9381	3.9362
Adapt.-rep.—2-period (LMS/256)	4.1765	4.0709	4.1276	3.9959	4.0223	4.0157
Adapt.-rep.—3-period (LMS/256)	3.9349	3.8114	3.9066	3.8053	3.8530	3.8871

disturbance rejection scheme discussed here. These are a simple LTI controller and a LTI minimum-variance-type controller, which was tuned using the inverse QR-RLS algorithm. The closed-loop plant, resulting from the interconnection of those controllers with the original open-loop plant of the HDD system, is G. A model of G, labeled as $\hat{G}$, was identified using the *n4sid* algorithm. For more details see Pérez-Arancibia et al. (2010) and Pérez Arancibia et al. (2007a,b).

5.2. Multiple-period repetitive controller design

The *power spectral density* (PSD) of the experimental PES y, obtained using the baseline controller shows that several sets of periodic signals are composing part of the baseline PES y. Besides the set of signals with frequencies multiple of 120 Hz, which is a direct consequence of the rotation of the HDD platters, there are sets of signals with frequencies multiple of 70, 407 Hz and others. In order to target these periodic signals in the PES, four repetitive controllers are designed, using the methodology described in Sections 2 and 3. The first controller is one-periodic repetitive designed to cancel signals with frequencies multiple of 120 Hz. The second controller is two-periodic repetitive designed to cancel signals with frequencies multiple of 70 and 120 Hz. The third controller is two-periodic repetitive designed to cancel signals with frequencies multiple of 120 and 407 Hz. And finally, the fourth controller is three-periodic repetitive designed to cancel signals with frequencies multiple of 70, 120 and 407 Hz.

In order to generate the notches that would allow us to cancel signals with the aforementioned frequencies, we pick internal models $D^{(1)}$, $D^{(2)}$ and $D^{(3)}$ with $N_1 = 78$, $N_2 = 134$ and $N_3 = 23$, respectively. Notice that $N_1 = 78$ generates notches with an exact period of 120 Hz. However, $N_2 = 134$ and $N_3 = 23$ generate notches with periods of 69.8507 Hz and 406.9565 Hz, respectively. The corresponding low-pass zero-phase filters q_1, q_2 and q_3 are given by

$$q_3 = q_2 = q_1 = \left(1 - 10^{-6}\right)\left[2\left(2q_0 - q_0^2\right) - \left(2q_0 - q_0^2\right)^2\right], \quad (32)$$

with

$$q_0(z^{-1}, z) = 0.2z^{-1} + 0.6 + 0.2z. \quad (33)$$

The corresponding parameters γ_1, γ_2 and γ_3 are given by $\gamma_3 = \gamma_2 = \gamma_1 = 4.5 \times 10^{-7}$.

The filters q_1, q_2, q_3 and the parameters γ_1, γ_2, γ_3 were chosen so that the stability condition (23) is satisfied, while achieving a reasonable good performance according to the frequency response of the sensitivity function ζ_o. The resulting sensitivity functions ζ_o, for all the cases considered here, are shown in Fig. 7. Since the resulting repetitive controllers are LTI, with the use of a model of the open-loop system and the baseline controllers, stability robustness can be analyzed using the classical indices *minimum gain margin* (MGM) and *minimum phase margin* (MPM). This analysis was done and in all cases MGM > 1.48 dB and |MPM| > 29.5 deg. The details are omitted for the sake of brevity.

5.3. Experimental results

In this subsection we show experimental results that provide evidence of the effectiveness of the proposed approach, using two sets of data. The first set of data was obtained at a specific location of the HDD (Head 1, Track 20,000), where several tests were performed in real time. These are summarized in Table 1 and Figs. 8–10. Fig. 8 compares the PSDs of the PES y for the cases in which the system is under baseline control (blue), three-period repetitive control (green), and three-period adaptive–repetitive control using the inverse QR-RLS algorithm with a filter Q of order 64 (red). There, it can be observed that the LTI three-period repetitive controller is capable of canceling the periodic spikes at frequencies multiple of 70, 120 and 407 Hz, while amplifying the inter-notch regions. This inter-notch amplification is canceled by the adaptive filter in the adaptive–repetitive scheme in Fig. 6. Details of this phenomenon can be observed in Fig. 10, which shows close-ups of a plot that compares the PSDs of the PESs y for the three experimental cases shown separately in Fig. 8. In order to clearly show the improvement in performance, Fig. 9 shows histograms comparing the PES y obtained with the use of the baseline controller and with the use of the adaptive–repetitive control scheme. Also using data obtained at location {Head 1, Track 20,000}, Table 1 compares the performances obtained using multiple-period repetitive controllers using various different parameters and in combination with the adaptive algorithms LMS and inverse QR-RLS.

The second set of data is summarized in Table 2. There, we show the performance index value 3σ for experiments performed at different locations of the HDD, employing the baseline controller, a LTI one-period repetitive controller (120 Hz), a LTI two-period repetitive controller (70, 120 Hz), a LTI three-period repetitive controller (70, 120, 407 Hz), and the adaptive–repetitive scheme in Fig. 6, using the LMS and inverse QR-RLS algorithms with filters Q of orders 64 and 256, respectively. Clearly, the effectiveness of the proposed method is demonstrated.

6. Conclusions

In this paper we presented a new method for synthesizing multiple-period repetitive controllers which are integrable into a minimum-variance control scheme that combines repetitive and adaptive components. The main result presented here is a theorem that states a particular solution to the multiple-period repetitive control problem. The theorem was proven using mathematical induction, and based on the proof, a period-recursive method for synthesizing multiple-period repetitive controllers was derived. Experimental results on the track-following control of a commercial hard disk drive were used to demonstrate the effectiveness of the proposed method, in which
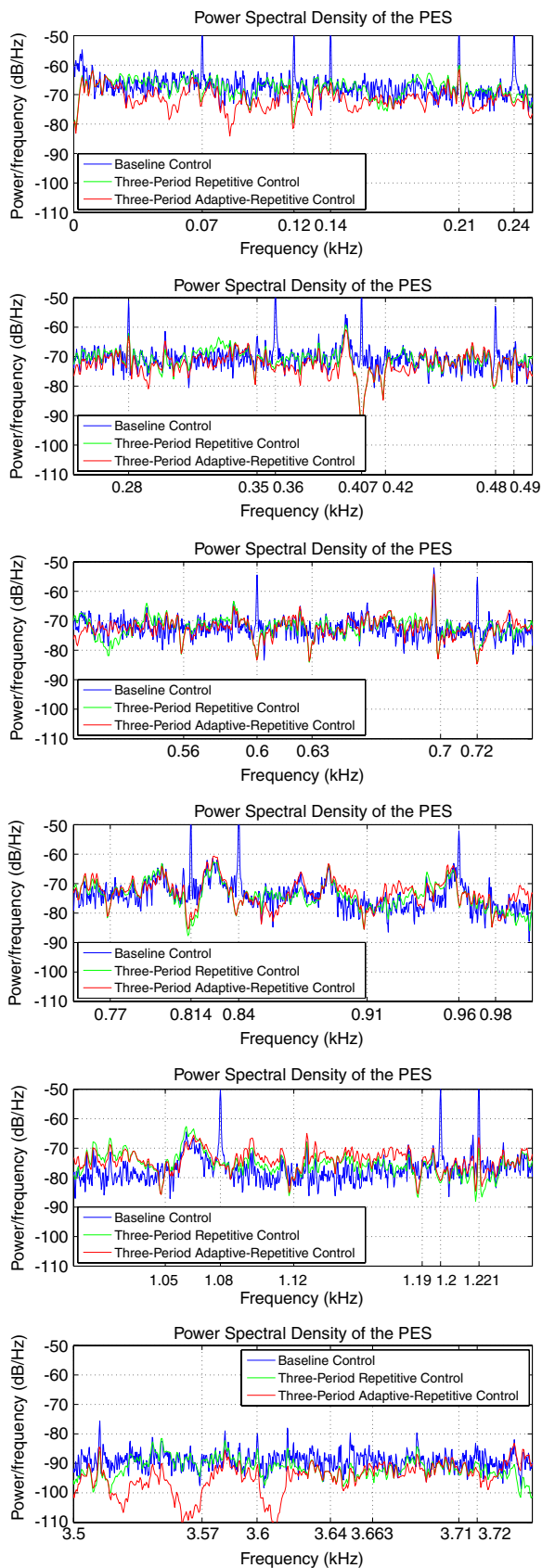
Fig. 10. Close-ups of a plot that compares the same PSDs of the PES *y* plotted in Fig. 8, obtained by three different methods.

a superior performance, evidenced by 3σ values of around 4% of the track width, is consistently achieved.

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